

Math 271
Limits and Their Properties

Find the limit of the following without a calculator:

1. $\lim_{x \rightarrow 1} \frac{x^2 - x}{x - 1}$

2. $\lim_{x \rightarrow 0} \frac{\sin 4x}{2x}$

3. $\lim_{x \rightarrow 2} \frac{x^2 + 3x - 10}{x^2 - 4}$

4. $\lim_{x \rightarrow 5} \frac{4 - \sqrt{1 + 3x}}{5 - x}$

5. $\lim_{x \rightarrow 1} \frac{x^2 - x}{x^3 - 1}$

6. $\lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{x+1}} - 1}{x}$

Find the vertical asymptotes for the following functions without a calculator:

7. $f(x) = \frac{4x - 1}{9 - x^2}$

8. $f(x) = \frac{x}{x^2 - 3x}$

9. $f(x) = \frac{x^2 - x - 6}{x - 3}$

Find the limits:

10. $\lim_{x \rightarrow 0^-} \sqrt{x}$

11. $\lim_{x \rightarrow 4^+} \sqrt{x}$

12. $\lim_{x \rightarrow -2} (x + 3)^{300}$

$$13. \lim_{x \rightarrow 0} \frac{2x - 1}{6x + 2}$$

$$14. \lim_{x \rightarrow 0} (x + \cos x)$$

$$15. \lim_{x \rightarrow \pi/2^-} (\tan x)$$

$$16. \lim_{x \rightarrow 2} \frac{1}{x - 2}$$

$$17. \lim_{x \rightarrow 4} \frac{x + 2}{x^2 + 6x + 8}$$

$$18. \lim_{x \rightarrow 0^+} f(x) = \begin{cases} x^3 & \text{for } x \geq 0 \\ 3x + 2 & \text{for } x < 0 \end{cases}$$

$$19. \lim_{x \rightarrow 1.5} \|x\|$$

$$20. \lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x) - 2x}{\Delta x}$$

$$21. \text{ If } \lim_{x \rightarrow 0} f(x) = 2 \text{ and } \lim_{x \rightarrow 0} g(x) = 5, \text{ what is } \lim_{x \rightarrow 0} [2f(x) - 3g(x)]$$

Determine if the function is continuous or discontinuous and tell where and what type of discontinuity occurs.

$$22. g(x) = \|x - 1\|$$

$$23. f(x) = 3x - 4$$

$$24. f(x) = \begin{cases} -2x & \text{if } x \leq 2 \\ x^2 - 4x + 1 & \text{if } x > 2 \end{cases}$$

Solve:

25. Determine the value of c such that the function $f(x)$ is continuous.

$$f(x) = \begin{cases} x + 3, & x \leq 2 \\ cx + 6, & x > 2 \end{cases}$$

26. State the Intermediate Value Theorem.

27. Use a table of numerical values to find $\lim_{x \rightarrow 1} f(x) = \frac{\sqrt{2x+1} - \sqrt{3}}{x-1}$

x	1.1	1.01	1.001	1.0001
f(x)				