

Math 271
Differentiation

Solve without a calculator:

1. Write one of the two definitions of a derivative of $f(x)$
2. $D_x(\sin x)$
3. $D_x(\cos x)$
4. $D_x(\tan x)$
5. $D_x(\cot x)$
6. $D_x(\sec x)$
7. $D_x(\csc x)$
8. What is the relationship between a function being continuous and a function being differentiable?

Calculate $\frac{dy}{dx}$ for the following functions. Leave no negative exponents.

9. $y = (8x^2 + 3x + 2)^5$

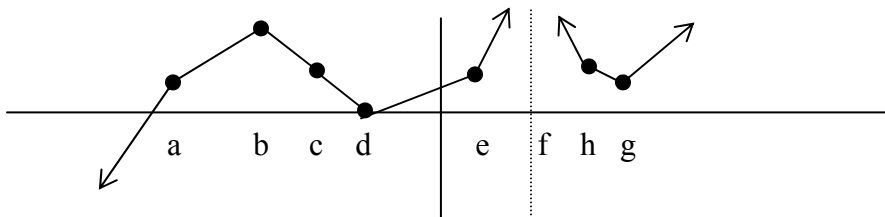
10. $y = \frac{x^3 + 4x^2 - 2x + 1}{x^2}$

11. $x^2 + y^2 + 3xy = 5$

12. $\sqrt{x+1}/x$

13. $y = x \sec^4(8x)$

Use the following graph to determine where the value of the derivative is:



14. Positive =

15. Negative =

16. Zero =

17. Undefined =

Solve:

18. A ladder is 25 feet long and is leaning against a house. The base of the ladder is sliding away from the house at a rate of 2ft. per second. How fast is the top of the ladder moving down the wall when the base of the ladder is 9ft. away from the wall?

19. Sand is falling into a conical pile at a rate of $10 \text{ ft}^3/\text{min}$. The diameter of the base of the cone is approximately three times the height. At what rate is the height of the pile changing when the pile is 15ft high?

20. A calculus instructor is thrown straight down (head first) from the top of a 1363ft tall building with an initial velocity of 30ft/sec.

- a. What is the position equation for this situation?
- b. What is the velocity equation for this situation?
- c. How long does it take for her to splat (hit the ground)?
- d. What is her average velocity for $t = 1$ to $t = 3$ sec?
- e. What is her final velocity?

21. Use the definition of the derivative (the long way) to find the derivative of $f(x) = 2x^2 + 5$.

22. What is the equation of the tangent line at $x = 2$ for the function $f(x) = x^3 + x^2 - 1$?

23. What is the rate of change of y with respect to x if $y = \sqrt{3x - 4}$ at $x = 5$?

(Extra Credit) A balloon rises at a rate of 2.5m/s from a point on the ground 50 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 50 meters above the ground.